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T. Vandellos, E. Martin, Dominique Leguillon. Comparison between cohesive zone models and a coupled criterion for prediction of edge debonding. 16th European Conference on Composite Materials, Jun 2014, SEVILLE, Spain. hal-01069269

**HAL Id: hal-01069269**

**<https://onera.hal.science/hal-01069269>**

Submitted on 29 Sep 2014

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## **COMPARISON BETWEEN COHESIVE ZONE MODELS AND A COUPLED CRITERION FOR PREDICTION OF EDGE DEBONDING**

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**Keywords:** Failure analysis, Finite Fracture Mechanics, Cohesive Zone Model.

### **Abstract**

*The onset of edge debonding within a bonded specimen submitted to bending is modeled with two numerical approaches: the coupled criterion and the cohesive zone model. The comparison of the results obtained with the both approaches evidences that (i) the prediction of edge debonding strongly depends on the shape of the cohesive law and (ii) the trapezoidal cohesive law is the most relevant model to predict the edge debonding as compared with the coupled criterion.*

### **1. Introduction**

The prediction of the onset of interface debonding is generally performed using a stress criterion [1] or linear fracture mechanics [2]. In each case, a characteristic length is needed which has to be identified experimentally. A first alternative approach is the use of a cohesive zone model [3] which simulates a progressive debonding build up in terms of continuum damage variables. It was shown by previous authors that the shape of the cohesive zone model (for a given value of the fracture toughness) does not have any influence on the steady state propagation of a rectilinear crack [4]. This result does not hold at crack initiation for which the shape of the cohesive law has a strong influence [4]. A second alternative approach is the coupled strength and energy criterion [5] which permits the prediction of the applied load involving the onset and the associated crack nucleation length. This approach has proved to be successful to analyze the onset of fracture mechanisms within composite materials [6,7,8] and bonded specimens [9,10]. An extension to the 3D geometry has been recently proposed [11] but, at the moment, the use of cohesive zone models seems to be easier.

The aim of this paper is to assess the capabilities of cohesive zone models to predict the failure onset of a bonded specimen submitted to bending as compared with the coupled criterion. In this study, it is important to note that the coupled criterion is just considered as a numerical reference solution; but this assumption should be experimentally verified. First, the both approaches are described. Second, several shapes of the cohesive law are compared to the coupled criterion in order to show their influence on the prediction. Finally, with the most relevant shape, the influence of the interfacial properties on the prediction of the edge debonding is presented to conclude about the use of the cohesive zone model.

## 2. The coupled criterion

As proposed by a previous author [5], combining an energy and a stress condition allows to derive an initiation criterion in the vicinity of a stress concentration. First, an energy balance between an elastic state prior to any crack growth and after the onset of a crack extension of area  $\delta S$  leads to the following incremental energy condition:

$$G_{inc}(a) = \frac{W(0) - W(a)}{\delta S} \geq G_C \quad (1)$$

where  $W(0)$  is the potential energy at the initial state (without crack),  $W(a)$  is the potential energy at the final state (with a crack of length  $a$ ) and  $G_{inc}$  is the incremental energy release rate in which the infinitesimal energy rates of the classical Griffith approach are replaced by finite energy increments.

Second, a stress condition states that the normal out-of-plane stress  $\sigma$  along the anticipated path of crack nucleation is greater than the relevant strength  $\sigma_C$

$$\sigma(x) \geq \sigma_C \quad \text{for } x \leq a \quad (2)$$

Finally, for a monotonic and increasing applied loading, the crack increment and the applied load at nucleation are obtained by combining the equations (1) and (2).

## 3. The cohesive zone models

Cohesive zone models are used to describe the behavior of interfaces. More precisely, the traction in mode I (*i.e.* the opening mode of fracture),  $T_1$  (resp.  $T_2$  in mode II (shearing mode)), between the top and bottom surfaces of the interface is related to the relative displacement in mode I  $\delta_1$  (resp.  $\delta_2$  in mode II). Several shapes of cohesive law (bilinear, trilinear and trapezoidal [12]), including an elastic part, can be obtained using the constitutive law written as

$$\begin{cases} \text{if } \delta_1 \geq 0 & \text{if } \delta_1 < 0 \\ \left\{ \begin{array}{l} T_1 = K\delta_1(1 - \lambda) \\ T_2 = K\delta_2(1 - \lambda) \end{array} \right. & \left\{ \begin{array}{l} T_1 = \alpha_C K\delta_1 \\ T_2 = K\delta_2(1 - \lambda) \end{array} \right. \end{cases} \quad (3)$$

where  $K$  is the initial stiffness of the interface,  $\alpha_C$  is a penalization factor for out-of-plane compression,  $\lambda$  is the damage variable, related to the damage kinetics, and  $f(\lambda)$  represents the effect of damage. The evolution of the damage variable  $\lambda$  is defined by

$$\left\{ \begin{array}{ll} \lambda = 0 & \text{if } \delta \leq \delta_0 \\ \lambda = (\delta_0 - \delta) \frac{\delta^* - \alpha_\sigma \delta_0}{\delta(\delta_0 - \delta^*)} & \text{if } \delta_0 < \delta \leq \delta^* \quad \text{and} \quad \dot{\lambda} \geq 0 \\ \lambda = \min \left\{ \frac{\delta(\delta^* - \delta_f) + \alpha_\sigma \delta_0(\delta_f - \delta)}{\delta(\delta^* - \delta_f)}, 1 \right\} & \text{if } \delta \geq \delta^* \end{array} \right. \quad (4)$$

The relative displacement  $\delta$  is determined by

$$\delta = \sqrt{\langle \delta_1 \rangle_+^2 + (\delta_2)^2} \quad (5)$$

where  $\langle \cdot \rangle_+$  are the classical Macaulay brackets defined by

$$\langle x \rangle_+ = \max\{0, x\} \quad (6)$$

It should be noted that, in order to avoid damage under pure out-of-plane compressive normal stress, the normal relative displacement  $\delta_1$  is only taken into account when positive. Parameters  $\delta_0$  and  $\delta_f$  are material constants corresponding respectively to the relative displacement associated with the damage threshold  $\sigma_C$  and the interfacial stiffness  $K$ , and to the relative displacement attained when the energy release rate  $G$  is equal to the fracture toughness  $G_C$ . The threshold  $\delta^*$  is the relative displacement associated with the maximal damageable stress  $\sigma^*$  which represents the admissible stress on the interface at the end of the first part of the damage process. It is defined by  $\sigma^* = \alpha_\sigma \sigma_C$  where  $\alpha_\sigma$  is a shape parameter. The value of  $\delta^*$  depends on the shape parameter  $\alpha_\delta$ . The couple  $(\alpha_\delta, \alpha_\sigma)$  defines where the negative slope of the cohesive law can be changed. Finally, the relative displacements  $\delta_0$ ,  $\delta^*$  and  $\delta_f$ , represented in Figure 1, are defined by

$$\begin{cases} \delta_0 = \sigma_C / K \\ \delta^* = \delta_0 + \alpha_\delta (\delta_f - \delta_0) \quad \text{with } \alpha_\delta = \frac{G_C - 0.5\sigma_C \delta_0}{0.5\sigma_C (\delta_f - \delta_0)} - \alpha_\sigma \\ \delta_f = \frac{G_C - 0.5\sigma_C \delta_0}{0.5\sigma_C (\alpha_\delta + \alpha_\sigma)} + \delta_0 \end{cases} \quad (7)$$

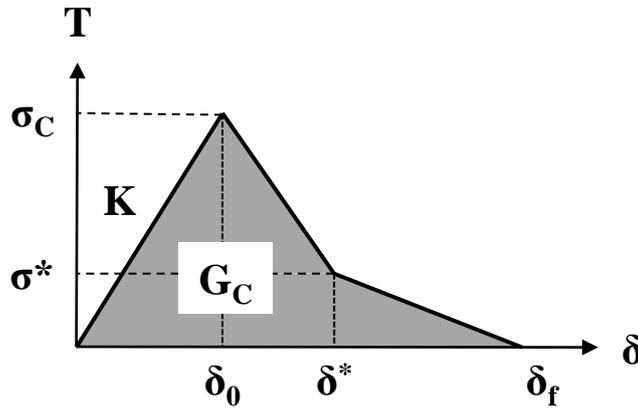


Figure 1. Representation of the relative displacements of the constitutive law from a tri-linear cohesive model

The shape of the cohesive law only depends on the values of the both shape parameters  $(\alpha_\delta, \alpha_\sigma)$  which are equal to  $(0,1)$  for the bilinear law and  $(\alpha_\delta, 1)$  for the trapezoidal one. The three laws are illustrated in Figure 2 for identical interfacial properties  $(\sigma_C, G_C)$ . In the next paragraphs, the value of the relative displacement  $\delta_0$  has been imposed to  $1.10^{-5}$  mm for all simulations. It is important to note that the parameter  $\delta_0$  is just considered as a numerical parameter. Moreover, two tri-linear shapes have been used, with  $(\alpha_\delta, \alpha_\sigma) = (0.05, 0.9)$  and

$(\alpha_\delta, \alpha_\sigma) = (0.3, 0.6)$ , and the shape parameter of the trapezoidal law has been imposed to  $\alpha_\delta = 0.9$ .

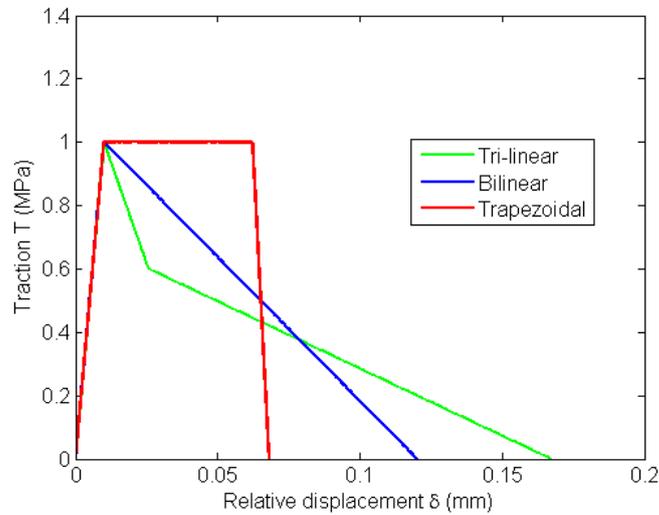


Figure 2. Illustration of the bilinear, tri-linear and trapezoidal shape of the cohesive zone model for the same interfacial properties

#### 4. Prediction of edge debonding: coupled criterion versus cohesive zone models

In order to compare the two numerical approaches, simulations of a bonded specimen submitted to four-point flexure loading have been performed. Cohesive zone models and the coupled criterion are here applied to analyze the initiation of fracture mechanisms near the free edge between the bond and the substrate. The geometry of the specimen is schematized in Figure 3. It consists of two substrates with the same thickness  $h = 2 \text{ mm}$  bonded with a thin interlayer which is here neglected. When the cohesive models are used, interface elements are inserted between the two bonded substrates. The elastic properties of the substrates are selected to be  $E_S = 400 \text{ GPa}$  (Young's modulus) and  $\nu_S = 0.2$  (Poisson's ratio). Due to the symmetry of the loaded specimen, it is assumed that the onset occurs near each free edge. A bidimensional finite element procedure with strongly refined mesh is used to derive the results which are now presented.

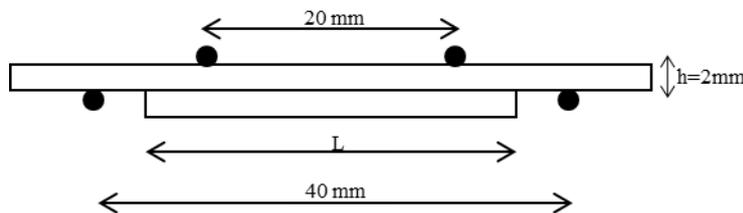


Figure 3. The geometry of the specimen tested under four-point bending

Several values of the interfacial strength  $\sigma_C$  and of the fracture toughness  $G_C$  have been studied for the comparison. These interfacial properties are considered similar whatever the fracture mode. The comparison between the load versus displacement curves obtained with the coupled criterion and with the cohesive zone models, for  $\sigma_C = 1 \text{ MPa}$ , (Figure 4) shows that the trapezoidal law is the most relevant model to predict the onset in a similar manner to

the coupled criterion, whatever the value of the fracture toughness. This observation is also true when  $\sigma_C = 10$  MPa.

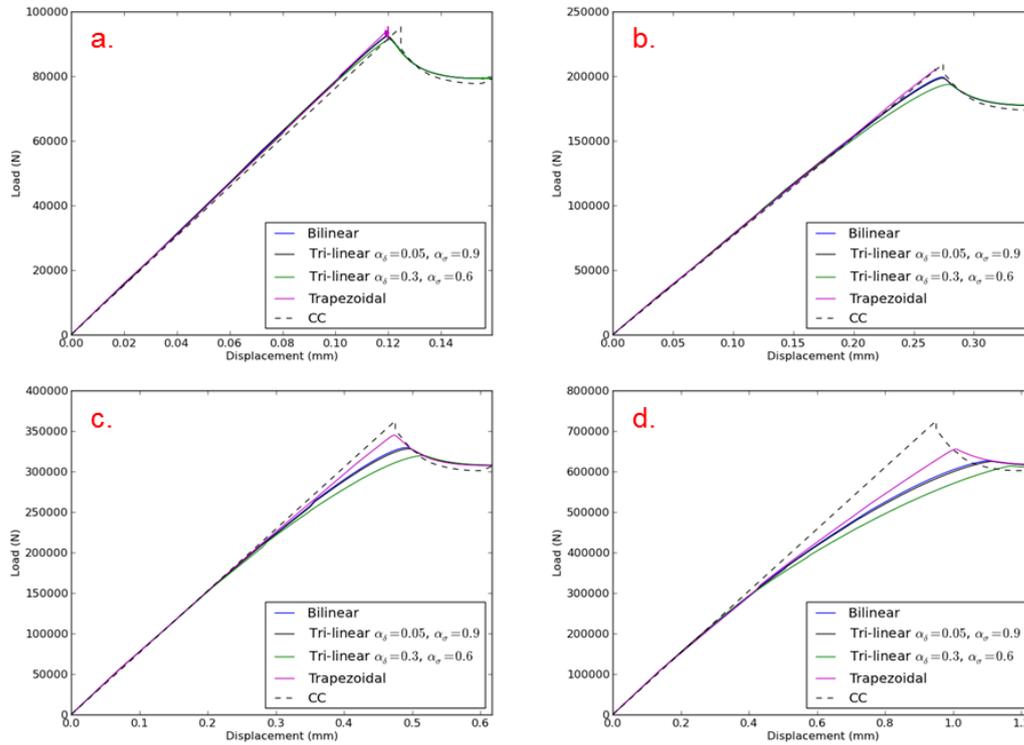


Figure 4. Comparison of load versus displacement curves between the coupled criterion (CC) and different cohesive zone models (CZM) with  $\sigma_C = 1$  MPa and (a.)  $G_C = 1$  J/m<sup>2</sup>, (b.)  $G_C = 5$  J/m<sup>2</sup>, (c.)  $G_C = 15$  J/m<sup>2</sup>, (d.)  $G_C = 60$  J/m<sup>2</sup>

This result can be explained comparing the damage kinetics of the different cohesive laws. Indeed, contrary to the bilinear and the tri-linear laws where the evolution of the damage variable  $\lambda$  between 0 (unbroken state) and 1 (broken state) is continuous and relatively slow, the damage variable of the trapezoidal model can evolve very quickly when the relative displacement  $\delta$  exceeds  $\delta^*$  (*i.e.* at the end of the plateau). This kinetics involves that the process zone (*i.e.* the area where the damage variable is positive but smaller than 1) with the trapezoidal law is smaller than the ones obtained with the bilinear and the tri-linear shapes, as shown in Figure 5. This observation could explain the better correlation obtained with the coupled criterion.

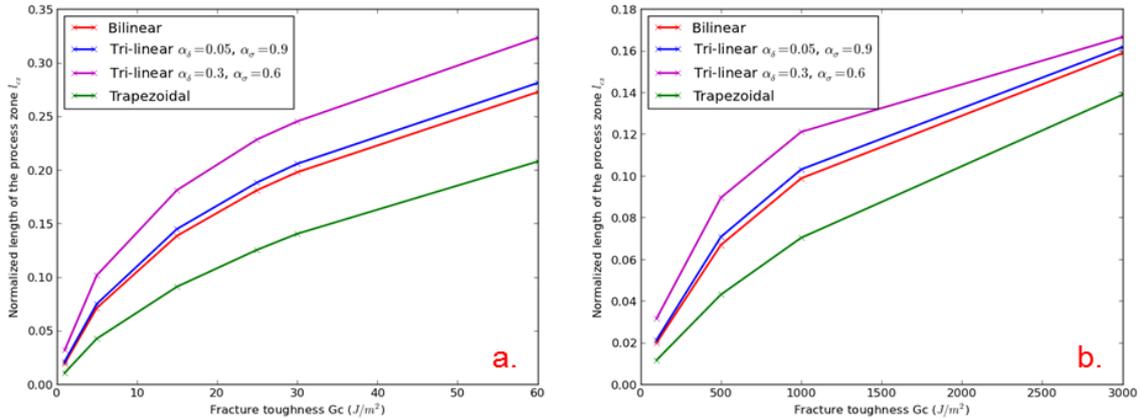


Figure 5. Normalized length of the process zone  $l_{cz}/L$  versus the fracture toughness  $G_C$  (a.) with  $\sigma_C = 1$  MPa and (b.) with  $\sigma_C = 10$  MPa.

Nevertheless, it is important to note that this correlation reduces when the fracture toughness increases (Figure 6 and Figure 7). This phenomenon results from the variation of the process zone length  $l_{cz}$ . Indeed, for the same interfacial strength  $\sigma_C$ , the larger the fracture toughness, the longer the length. Thus, the softening behavior becomes noticeable when the normalized crack length  $l_{cz}/L$  is higher than 10%. Consequently, as indicated in Table 1 and Table 2, the percentages error of the fracture displacement  $d_c$  and of the fracture load  $F_c$  exceed 5% as the consequence of softening (*i.e.* when  $G_C > 15$  J/m<sup>2</sup> for  $\sigma_C = 1$  MPa and when  $G_C > 1000$  J/m<sup>2</sup> for  $\sigma_C = 10$  MPa).

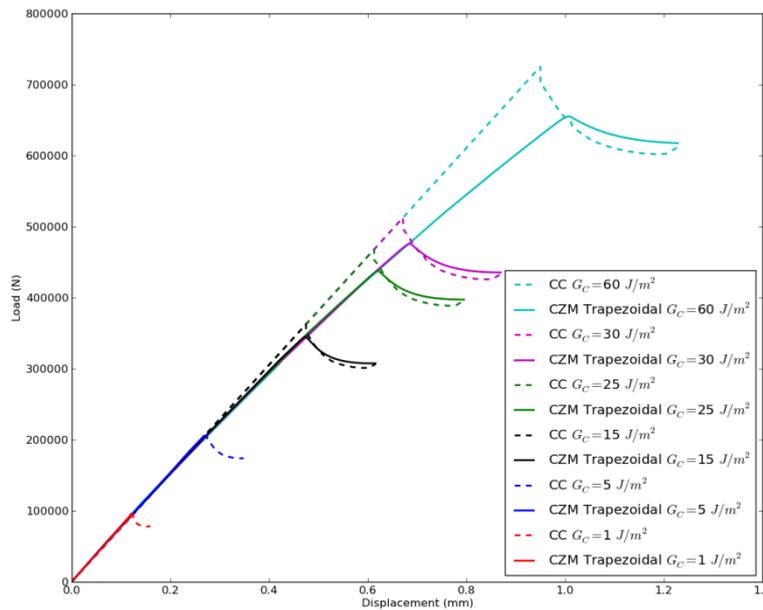


Figure 6. Comparison of load versus displacement curves between the coupled criterion (CC) and the trapezoidal cohesive zone model (CZM) for several fracture toughness with  $\sigma_C = 1$  MPa

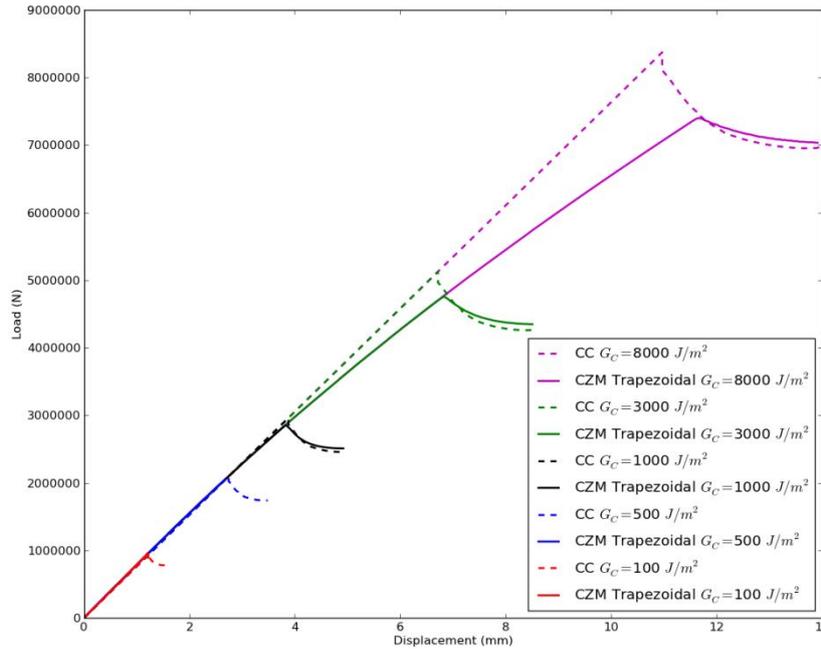


Figure 7. Comparison of load versus displacement curves between the coupled criterion (CC) and the trapezoidal cohesive zone model (CZM) for several fracture toughness with  $\sigma_c = 10$  MPa

$G_c$ (J/m <sup>2</sup> )	1	5	15	25	30	60
$100 \left( \frac{d_c^{CC} - d_c^{CZM}}{d_c^{CC}} \right)$	4.7	2.4	-0.08	-1.5	-2.7	-6.3
$100 \left( \frac{F_c^{CC} - F_c^{CZM}}{F_c^{CC}} \right)$	1.8	2.3	4.9	6.6	7.5	9.7

Table 1. Percentages error of the fracture displacement  $d_c$  and the fracture load  $F_c$  obtained by the trapezoidal cohesive zone model with  $\sigma_c = 1$  MPa

$G_c$ (J/m <sup>2</sup> )	100	500	1000	3000	8000
$100 \left( \frac{d_c^{CC} - d_c^{CZM}}{d_c^{CC}} \right)$	4.7	2.4	1	-1.8	-6.4
$100 \left( \frac{F_c^{CC} - F_c^{CZM}}{F_c^{CC}} \right)$	1.8	2.2	3.8	7.2	11.6

Table 2. Percentages error of the fracture displacement  $d_c$  and the fracture load  $F_c$  obtained by the trapezoidal cohesive zone model with  $\sigma_c = 10$  MPa

## 5. Conclusion

Simulations of a four-point bending test, for the prediction of edge debonding, with a coupled criterion and several cohesive zone models have been realized. First, the influence of the shape of the cohesive law on the prediction of the onset has been shown. The trapezoidal model appears the most adapted model to predict the initiation in a similar manner to the coupled criterion. Second, it has been demonstrated that the length of the process zone has to be small enough to verify a good correlation between the results obtained by the both

numerical approaches. Therefore, it seems possible to use the trapezoidal cohesive zone model, under a few material conditions, to predict precisely the edge debonding. A comparison between numerical and experimental results will be realized in future works in order to confirm the relevance of the trapezoidal cohesive zone model.

## **References**

- [1] J. C. Brewer and P. A. Lagace. Quadratic stress criterion for initiation of delamination. *Journal of composite materials*, volume(22):1141-1155, 1988.
- [2] M. E. Waddoups, J. R. Eisenmann and B. E. Kaminski. Macroscopic fracture mechanics of advanced composite materials. *Journal of composite materials*, volume(5):446-454, 1971.
- [3] A. Needleman. A continuum model for void nucleation by inclusion debonding. *Journal of Applied Mechanics*, volume(54):525-531, 1987.
- [4] M. Jean, V. Acary and Y. Monerie. Non-smooth contact dynamics approach of cohesive materials. *Philosophical Transactions of the Royal Society A*, volume(359):2497-2518, 2001.
- [5] D. Leguillon. Strength or toughness? A criterion for crack onset at a notch. *European Journal of Mechanics A/Solids*, volume(21):61-72, 2002.
- [6] V. Mantic. Interface crack onset at a circular cylindrical inclusion under a remote transverse tension. Application of a coupled stress and energy criterion. *International journal of Solids and Structures*, volume(46):1287-1304, 2009.
- [7] E. Martin, D. Leguillon and N. Carrère. A twofold strength and toughness criterion for the onset of free-edge shear delamination in angle-ply laminates. *International journal of Solids and Structures*, volume(47):1297-1305, 2010.
- [8] E. Martin, D. Leguillon and N. Carrère. A coupled strength and toughness criterion for the prediction of the open-hole tensile strength of a composite plate. *International journal of Solids and Structures*, volume(49):3915-3922, 2012.
- [9] A. Moradi, N. Carrère, D. Leguillon, E. Martin and J. Y. Cognard. Strength Prediction of Bonded Assemblies Using a Coupled Criterion under Elastic Assumptions: Effect of Material and Geometrical Parameters. *International Journal of Adhesion and Adhesives*, volume(47):73-82, 2013.
- [10] L. M. Nguyen, D. Leguillon, O. Gillia and E. Riviere. Bond failure of a SiC/SiC brazed assembly. *Mechanics of Materials*, volume(50):1-8, 2012.
- [11] D. Leguillon. An attempt to extend the 2D coupled criterion for crack nucleation in brittle materials to the 3D case. *Submitted*, 2014.
- [12] T. Vandellos, C. Huchette and N. Carrère. Proposition of a framework for the development of a cohesive zone model adapted to Carbon-Fiber Reinforced Plastic laminated composites. *Composite Structures*, volume(105):199-206, 2013.